Day 22

Slopes of Parallel Lines

If m_1 and m_2 represent the slopes of two parallel (nonvertical) lines, then

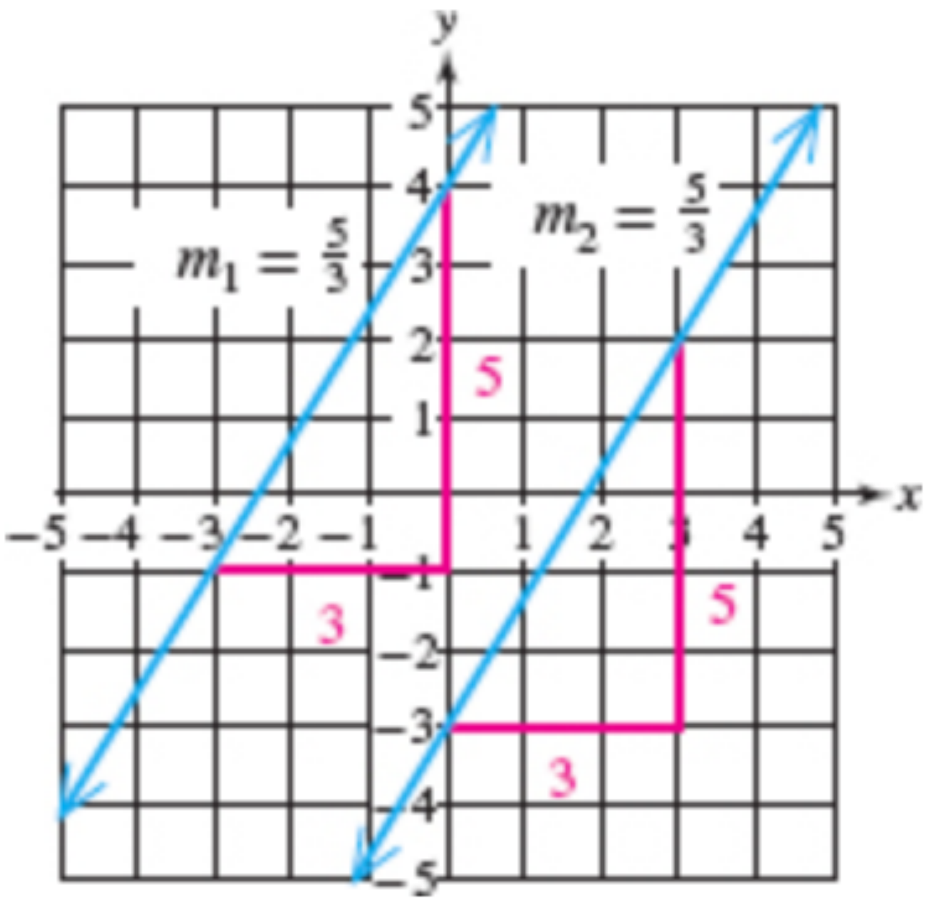
$$m_1 = m_2.$$

See Figure 3-23.

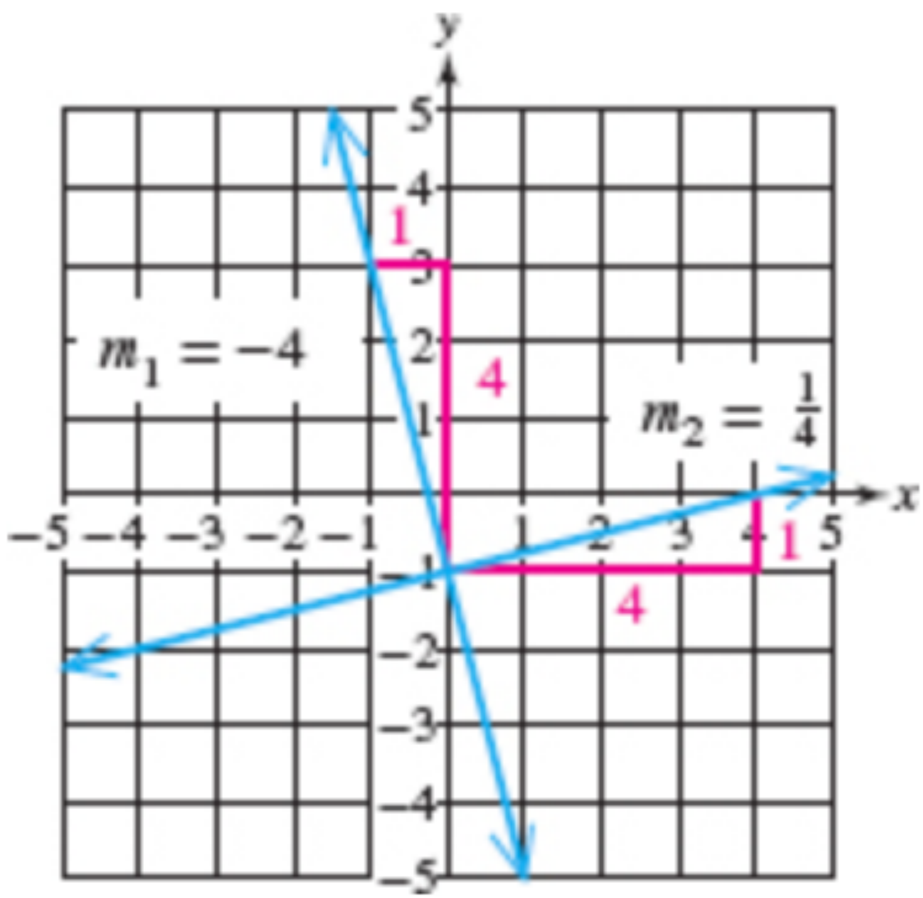
Slopes of Perpendicular Lines

If $m_1 \neq 0$ and $m_2 \neq 0$ represent the slopes of two perpendicular lines, then

$m_1 = -\dfrac{1}{m_2}$ or equivalently, $m_1 m_2 = -1$. See Figure 3-24.



y
x
$m_1 = \frac{5}{3}$
$m_2 = \frac{5}{3}$
5
3
5
3
−5 −4 −3 −2 −1 1 2 3 4 5
5 4 3 2 1 −1 −2 −3 −4 −5



y
x
$m_1 = -4$
$m_2 = \frac{1}{4}$
1
4
4
1

Lines l_1 and l_2 pass through the given points. Determine if l_1 and l_2 are parallel, perpendicular, or neither.

$$l_1: (2, -7) \text{ and } (4, 1) \quad l_2: (-3, 1) \text{ and } (1, 0)$$

Solution:

Find the slope of each line.

$$l_1: \quad \underset{(x_1, y_1)}{(2, -7)} \quad \text{and} \quad \underset{(x_2, y_2)}{(4, 1)}$$

$$m_1 = \frac{1 - (-7)}{4 - 2}$$

$$m_1 = \frac{8}{2}$$

$$m_1 = 4$$

$$l_2: \quad \underset{(x_1, y_1)}{(-3, 1)} \quad \text{and} \quad \underset{(x_2, y_2)}{(1, 0)}$$

$$m_2 = \frac{0 - 1}{1 - (-3)}$$

$$m_2 = \frac{-1}{4}$$

$$m_2 = -\frac{1}{4}$$

ProTip

TIP: You can check that two lines are perpendicular by checking that the product of their slopes is −1.

$$4\left(-\frac{1}{4}\right) = -1$$

Slope Intercept form

$mx + b = y$ or $y = mx + b$ The equation is in slope-intercept form.

Given the equation $-5x - 2y = 6$,

a. Write the equation in slope-intercept form.

b. Identify the slope and y-intercept.

Solution:

a. Write the equation in slope-intercept form, $y = mx + b$, by solving for y.

$$-5x - 2y = 6$$

$$-2y = 5x + 6 \qquad \text{Add } 5x \text{ to both sides.}$$

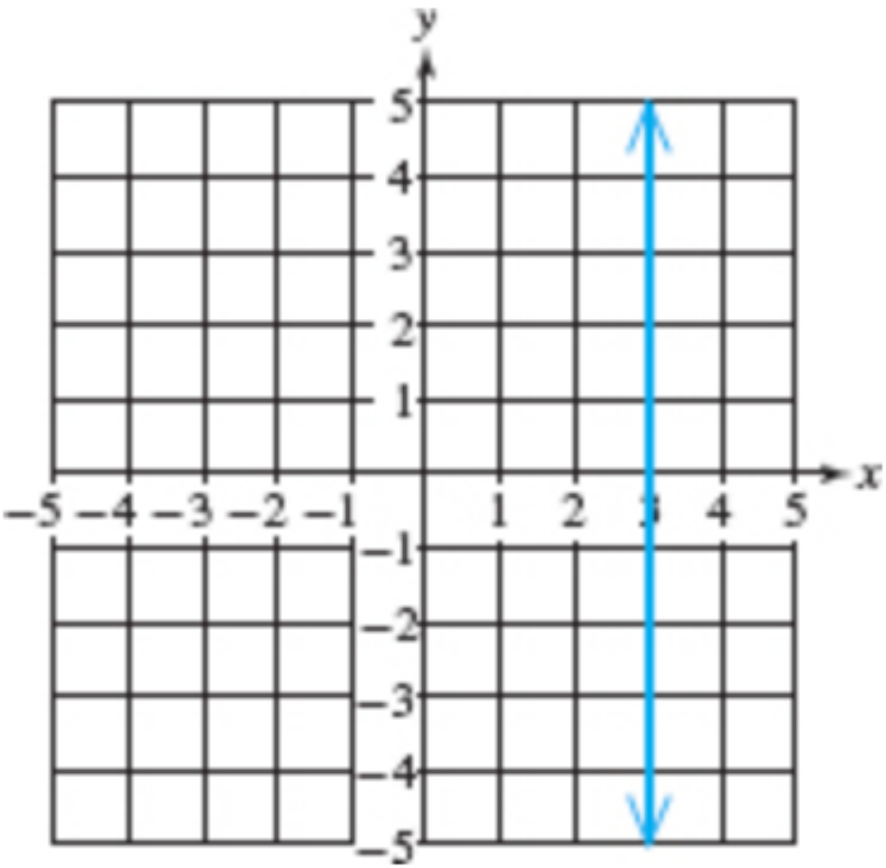
$$\frac{-2y}{-2} = \frac{5x+6}{-2} \qquad \text{Divide both sides by } -2.$$

$$y = \frac{5x}{-2} + \frac{6}{-2} \qquad \text{Divide each term by } -2 \text{ and simplify.}$$

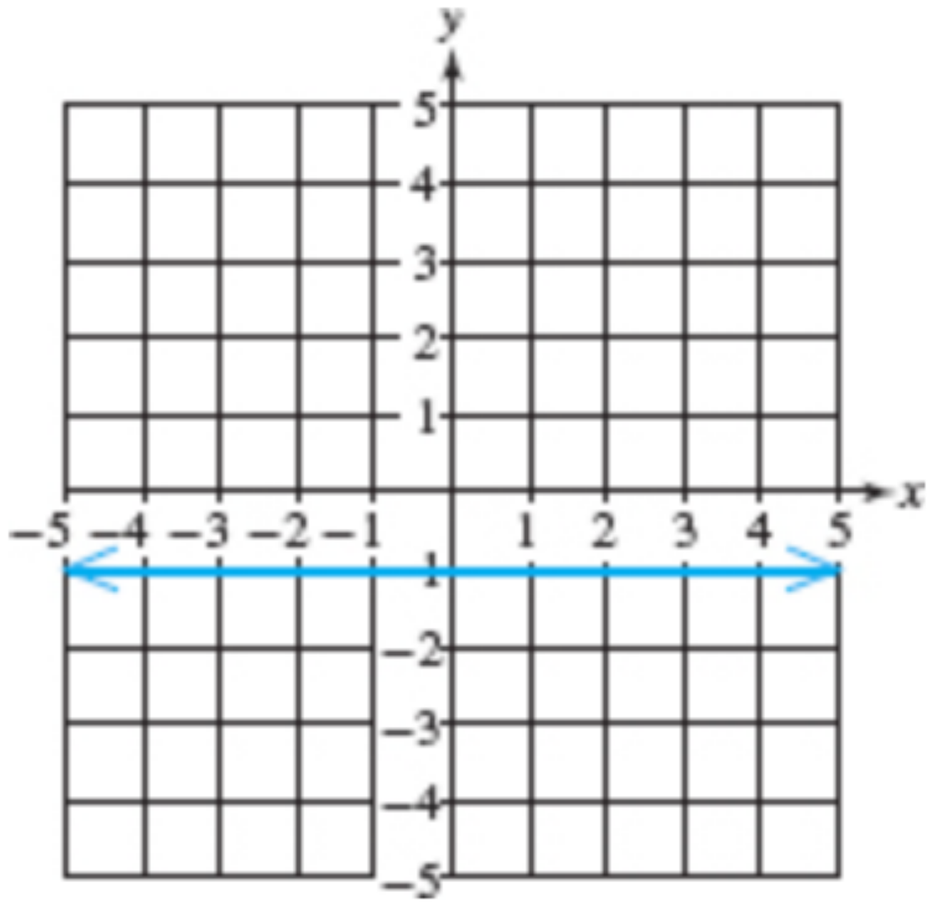
$$y = -\frac{5}{2}x - 3 \qquad \text{Slope-intercept form}$$

b. The slope is $-\frac{5}{2}$, and the y-intercept is (0,–3).

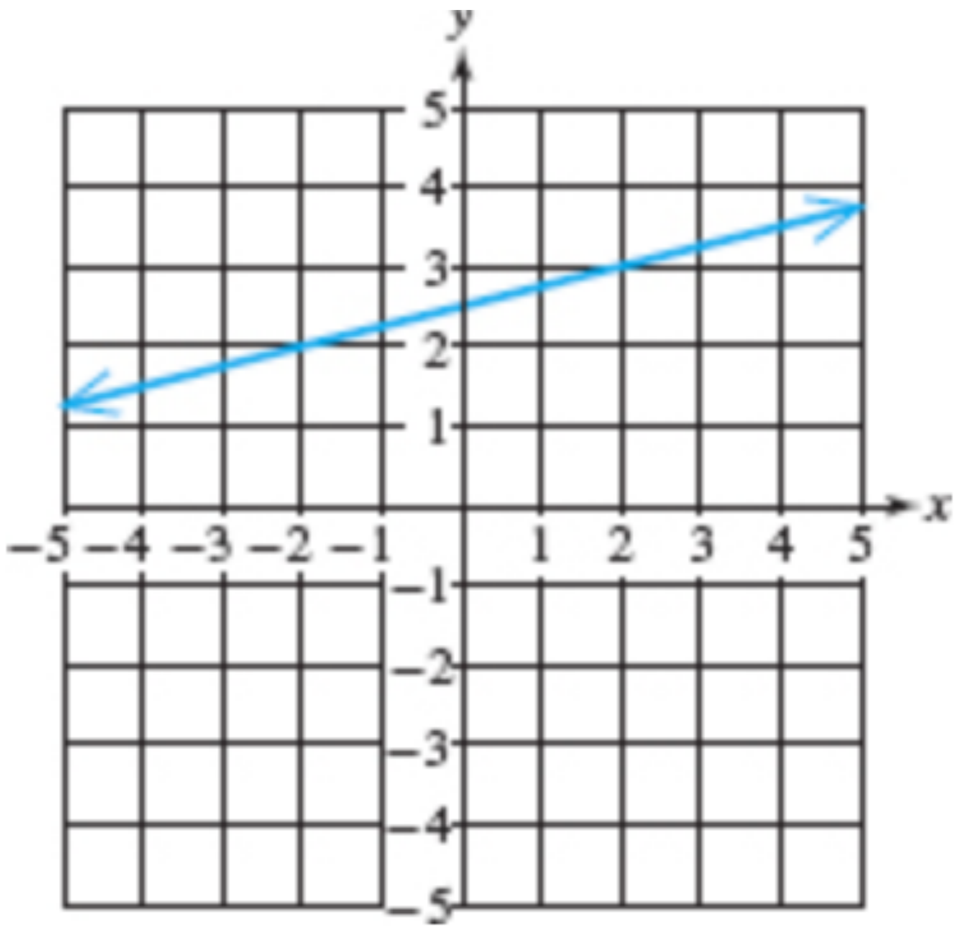
What's the slope?



What's the Slope?



What's the Slope?



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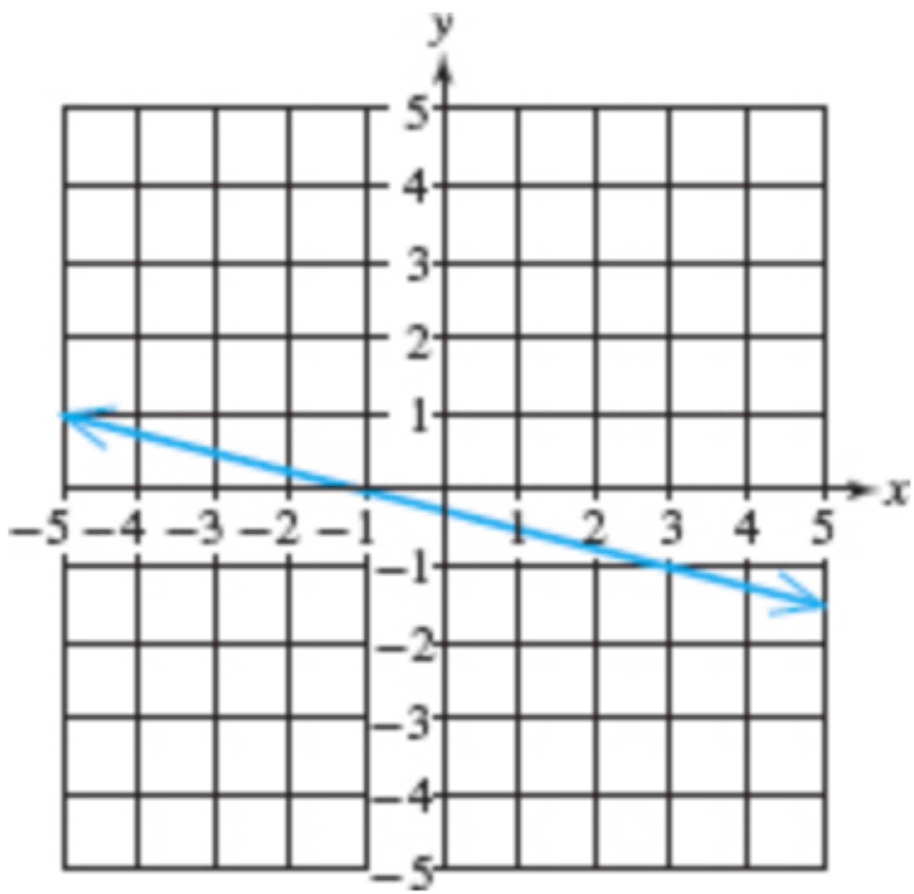
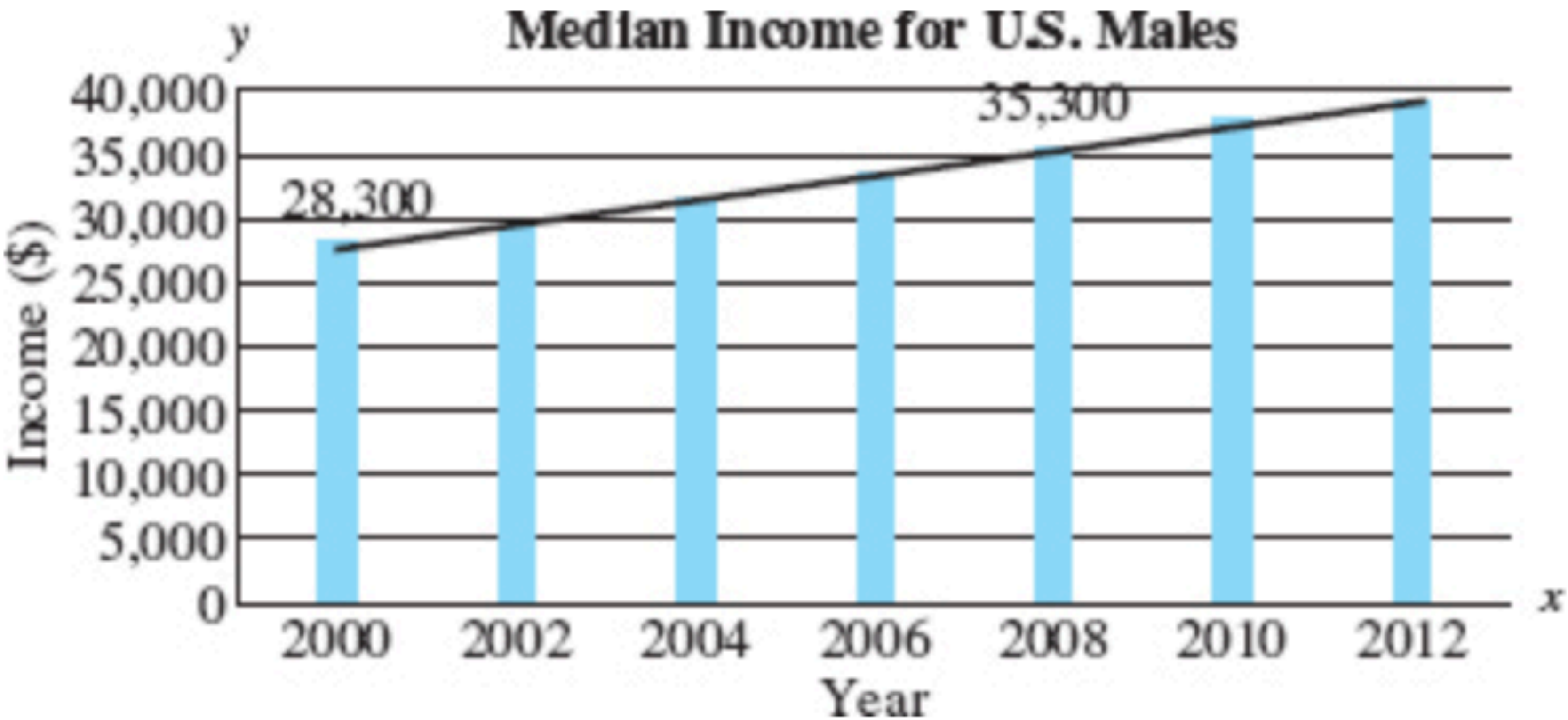
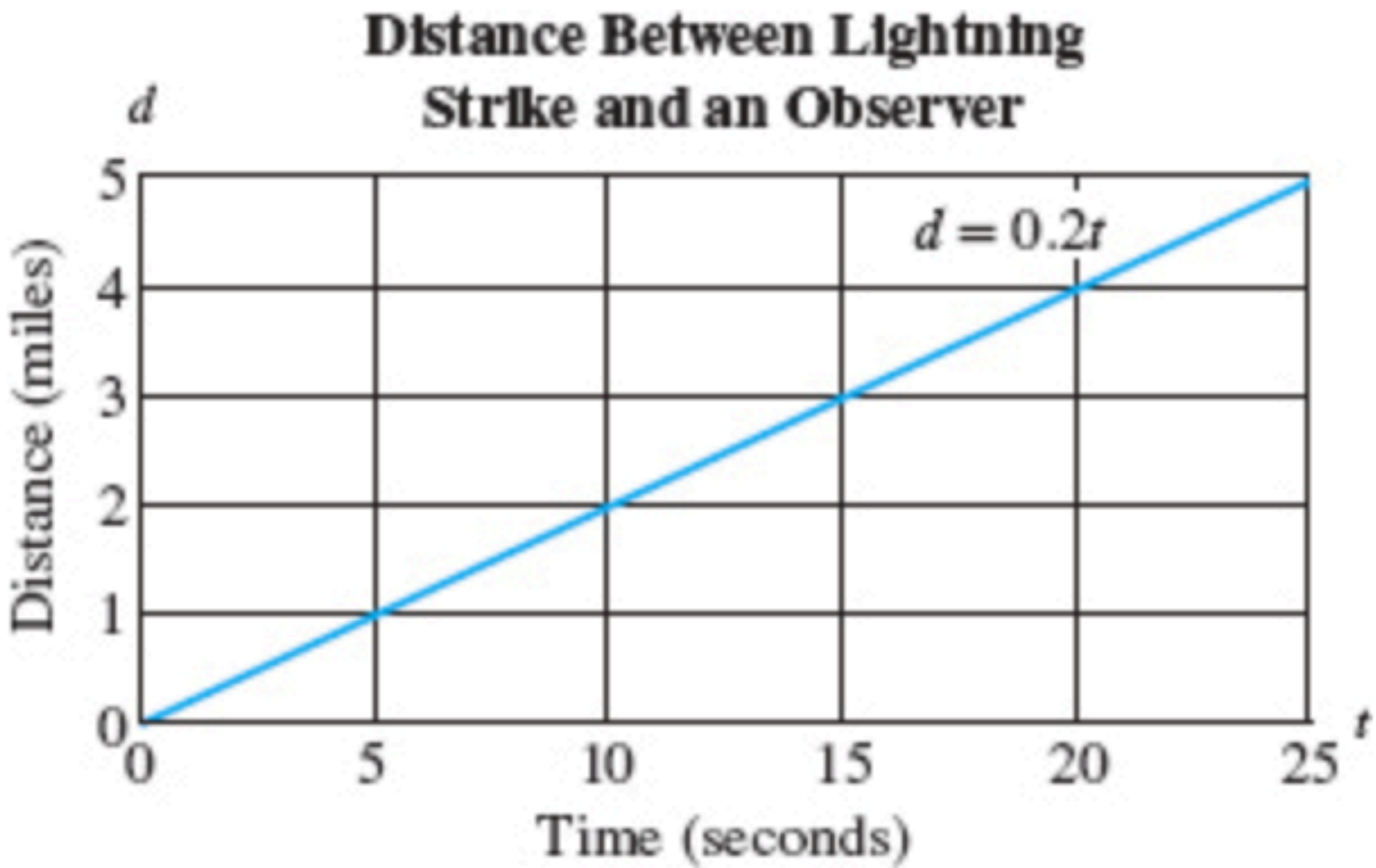


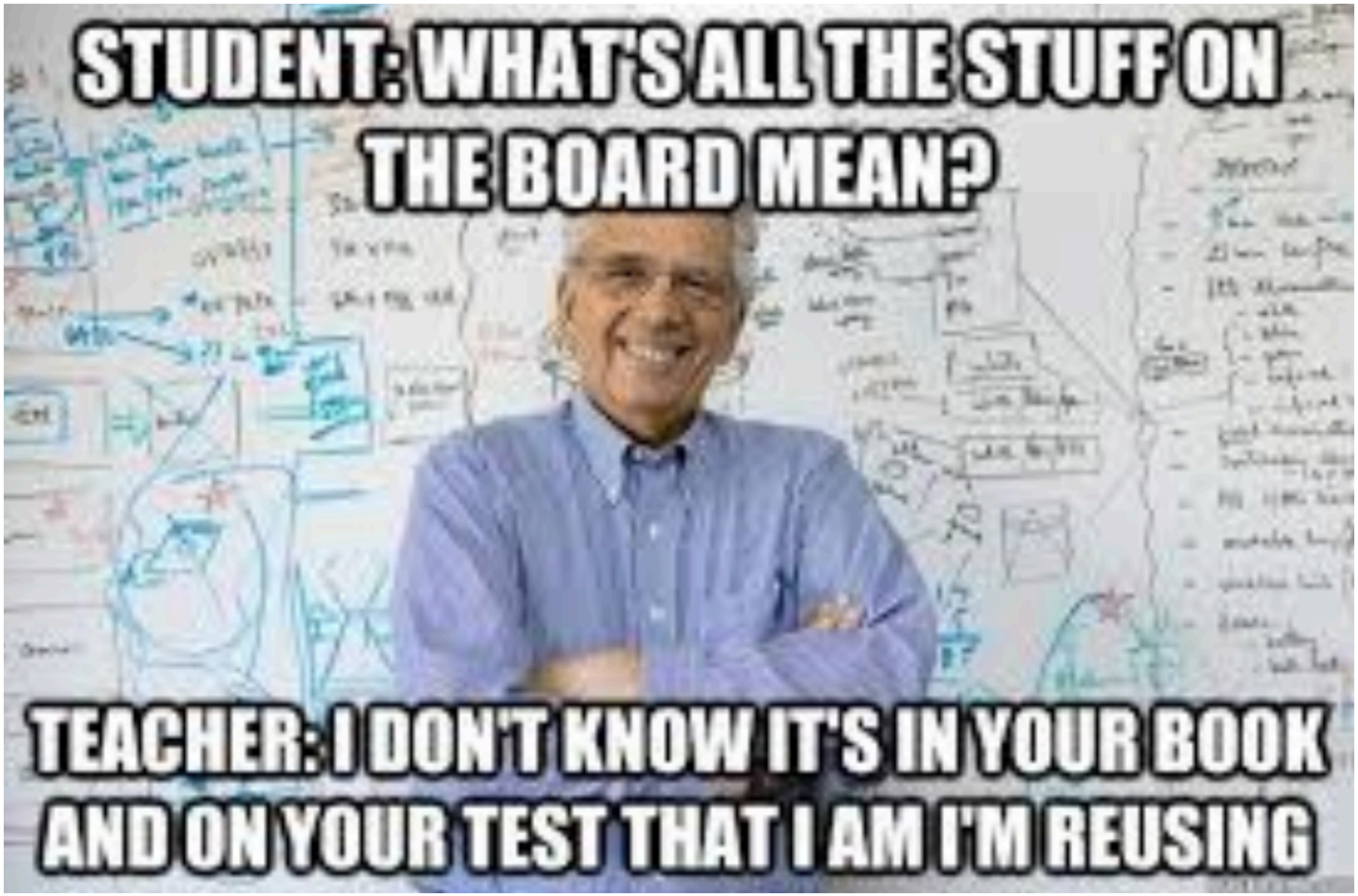
What's the Slope?



What's the Slope?



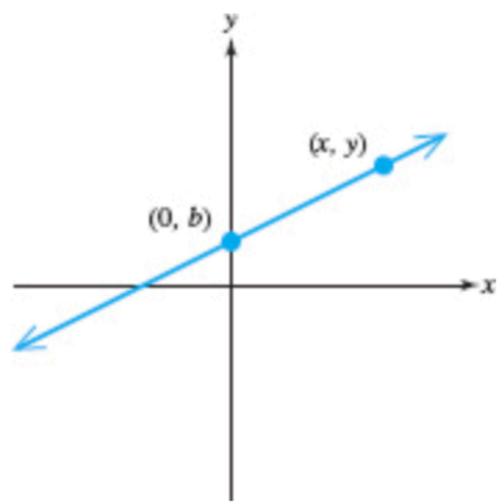
But what is this all for?



Slope and the Linear Equation:

Let $(0, b)$ represent (x_1, y_1), and let (x, y) represent (x_2, y_2). Apply the slope formula.

$m = \frac{y_2 - y_1}{x_2 - x_1} \rightarrow m = \frac{y - b}{x - 0}$	Apply the slope formula.
$m = \frac{y - b}{x}$	Simplify.
$mx = \left(\frac{y - b}{x}\right)x$	Multiply by x to clear fractions.
$mx = y - b$	
$mx + b = y - b + b$	To isolate y, add b to both sides.
$mx + b = y$ or $y = mx + b$	The equation is in slope-intercept form.



Solve these as a class

For each equation, identify the slope and y-intercept.

a. $y = 3x - 1$

b. $y = -2.7x + 5$

c. $y = 4x$

Solution:

Each equation is written in slope-intercept form, $y = mx + b$. The slope is the coefficient of x, and the y-intercept is determined by the constant term.

a. $y = 3x - 1$ The slope is 3. The y-intercept is $(0,-1)$.

b. $y = -2.7x + 5$ The slope is -2.7. The y-intercept is $(0,5)$.

c. $y = 4x$ can be written as $y = 4x + 0$. The slope is 4.
The y-intercept is $(0,0)$.

Slope-Intercept Form of a Linear Equation

$y = mx + b$ is the slope-intercept form of a linear equation. m is the slope and the point $(0, b)$ is the y-intercept.

Solve this on your own:

Given the equation $-5x - 2y = 6$,

a. Write the equation in slope-intercept form.

b. Identify the slope and *y*-intercept.

Solution:

a. Write the equation in slope-intercept form, $y = mx + b$, by solving for y.

$$-5x - 2y = 6$$

$$-2y = 5x + 6 \qquad \text{Add } 5x \text{ to both sides.}$$

$$\frac{-2y}{-2} = \frac{5x+6}{-2} \qquad \text{Divide both sides by } -2.$$

$$y = \frac{5x}{-2} + \frac{6}{-2} \qquad \text{Divide each term by } -2 \text{ and simplify.}$$

$$y = -\frac{5}{2}x - 3 \qquad \text{Slope-intercept form}$$

b. The slope is $-\frac{5}{2}$, and the y-intercept is $(0,-3)$.

Graph the equation $y = -\frac{5}{2}x - 3$ by using the slope and y-intercept.

Solution:

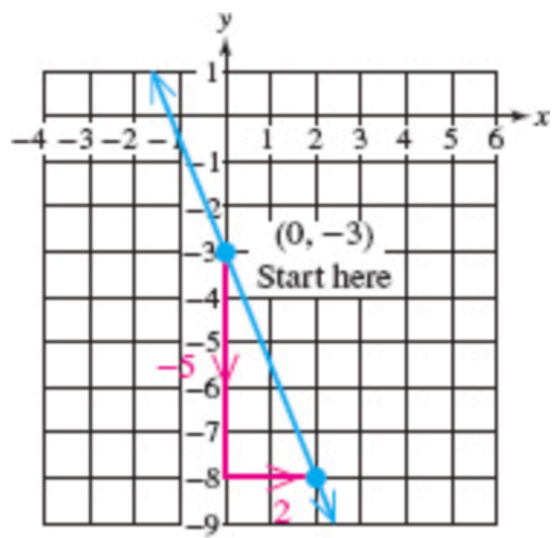
First plot the *y*-intercept, (0, –3).

The slope $m = -\frac{5}{2}$ can be written as

The slope $m = -\frac{5}{2}$ can be written as

$$m = \frac{-5}{2} \begin{array}{l} \longleftarrow \text{The change in } y \text{ is } -5. \\ \longleftarrow \text{The change in } x \text{ is } 2. \end{array}$$

To find a second point on the line, start at the *y*-intercept and move down 5 units and to the right 2 units. Then draw the line through the two points (**Figure 3-27**).



Do this on your own:

For each pair of lines, determine if they are parallel, perpendicular, or neither.

a. l_1: $y = 3x - 5$

l_2: $y = 3x + 1$

b. l_1: $y = \frac{3}{2}x + 2$

l_2: $y = \frac{2}{3}x + 1$

Hint:

Slopes of Parallel Lines

If m_1 and m_2 represent the slopes of two parallel (nonvertical) lines, then

$$m_1 = m_2.$$

See **Figure 3-23**.

Slopes of Perpendicular Lines

If $m_1 \neq 0$ and $m_2 \neq 0$ represent the slopes of two perpendicular lines, then

$$m_1 = -\frac{1}{m_2} \text{ or equivalently, } m_1 m_2 = -1.$$ See **Figure 3-24**.

Solution:

a. $l_1: y = 3x - 5$ The slope of l_1 is 3.

$l_2: y = 3x + 1$ The slope of l_2 is 3.

Because the slopes are the same, the lines are parallel.

b. $l_1: y = \frac{3}{2}x + 2$ The slope of l_1 is $\frac{3}{2}$.

$l_2: y = \frac{2}{3}x + 1$ The slope of l_2 is $\frac{2}{3}$.

The slopes are not the same. Therefore, the lines are not parallel. The values of the slopes are reciprocals, but they are not opposite in sign. Therefore, the lines are not perpendicular. The lines are neither parallel nor perpendicular.

Solve as a Class:

For each pair of lines, determine if they are parallel, perpendicular, or neither.

a. $l_1: x - 3y = -9$

$l_2: 3x = -y + 4$

b. $l_1: x = 2$

$l_2: 2y = 8$

Solution pt. A

Solution:

a. First write the equation of each line in slope-intercept form.

l_1: $x - 3y = -9$

$$-3y = -x - 9$$

$$\frac{-3y}{-3} = \frac{-x}{-3} - \frac{9}{-3}$$

$$y = \frac{1}{3}x + 3$$

l_2: $3x = -y + 4$

$$3x + y = 4$$

$$y = -3x + 4$$

l_1: $y = \frac{1}{3}x + 3$ The slope of l_1 is $\frac{1}{3}$.

l_2: $y = -3x + 4$ The slope of l_2 is -3.

The slope of $\frac{1}{3}$ is the opposite of the reciprocal of –3. Therefore, the lines are perpendicular.

Solution pt. B

b. The equation $x = 2$ represents a vertical line because the equation is in the form $x = k$.

The equation $2y = 8$ can be simplified to $y = 4$, which represents a horizontal line.

In this example, we do not need to analyze the slopes because vertical lines and horizontal lines are perpendicular.

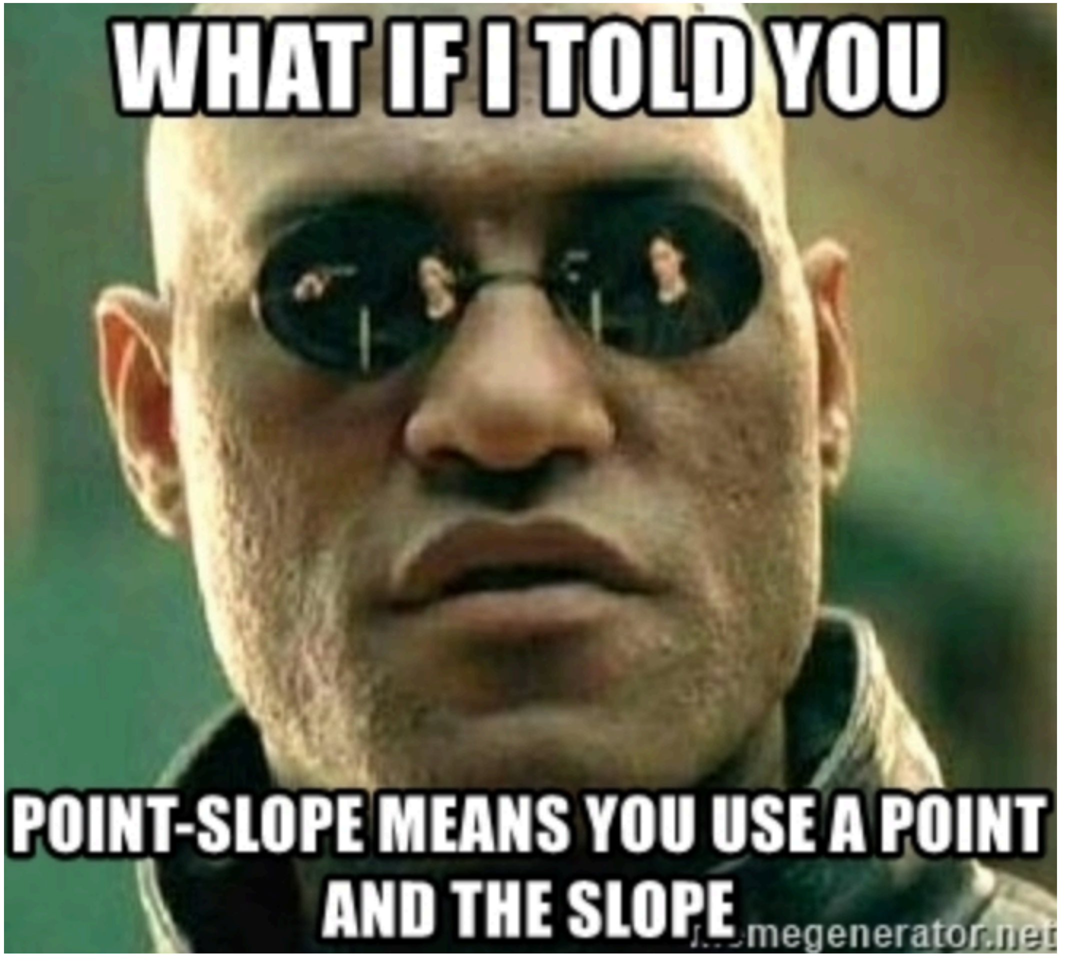
Write an equation of the line whose slope is $\frac{2}{3}$ and whose *y*-intercept is (0, 8).

Solution:

The slope is given as $m = \frac{2}{3}$, and the y-intercept $(0, b)$ is given as $(0, 8)$. Substitute the values $m = \frac{2}{3}$ and $b = 8$ into the slope-intercept form of a line.

$$y = mx + b$$
$$y = \frac{2}{3}x + 8$$

Point-Slope Form



Suppose a line passes through a given point (x_1, y_1) and has slope m. If (x, y) is any other point on the line, then the slope is given by

$$m = \frac{y - y_1}{x - x_1} \qquad \text{Slope formula}$$

$$m(x - x_1) = \frac{y - y_1}{\cancel{x - x_1}}\cancel{(x - x_1)} \qquad \text{Clear fractions.}$$

$$m(x - x_1) = y - y_1$$

$$y - y_1 = m(x - x_1) \qquad \text{Point-slope formula}$$

Point-Slope Formula

The **point-slope formula** is given by

$$y - y_1 = m(x - x_1)$$

where m is the slope of the line and (x_1, y_1) is any known point on the line.

Solve as a class:

Use the point-slope formula to write an equation of the line having a slope of 3 and passing through the point (–2, –4). Write the answer in slope-intercept form.

Solution:

The slope of the line is given: $m = 3$

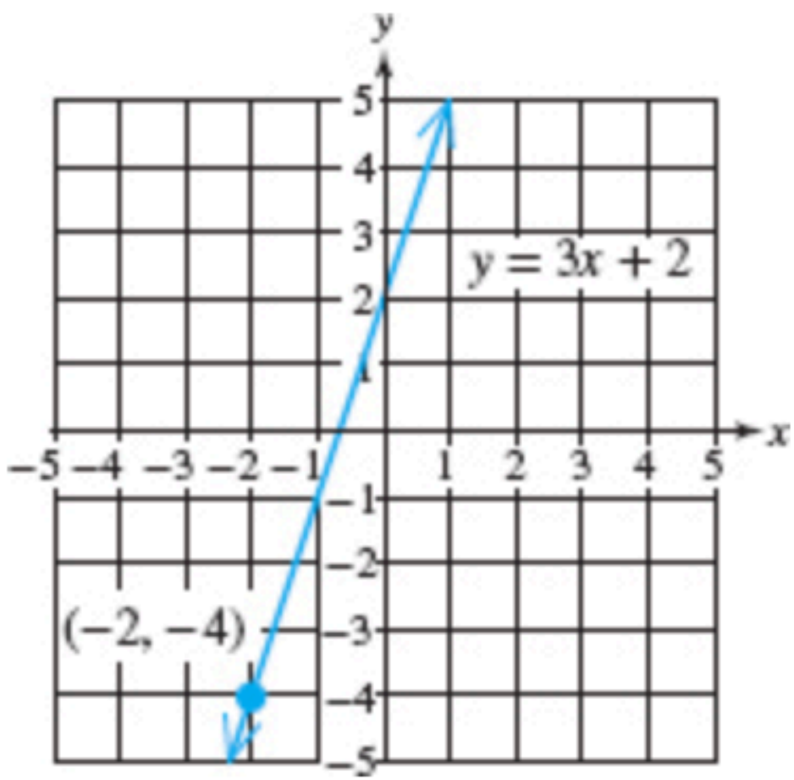
A point on the line is given: $(x_1, y_1) = (-2, -4)$

The point-slope formula:

$y - y_1 = m(x - x_1)$	
$y - (-4) = 3[x - (-2)]$	Substitute $m = 3$, $x_1 = -2$, and $y_1 = -4$.
$y + 4 = 3(x + 2)$	Simplify. Because the final answer is required in slope-intercept form, simplify the equation and solve for y.
$y + 4 = 3x + 6$	Apply the distributive property.
$y = 3x + 2$	Slope-intercept form

Let's graph it!

Notice that the line does indeed pass through the point (–2, –4).



Solve as a class:

Use the point-slope formula to find an equation of the line passing through the points $(-2, 5)$ and $(4, -1)$. Write the final answer in slope-intercept form.

Solution:

Given two points on a line, the slope can be found with the slope formula.

$$\underset{(x_1, y_1)}{(-2, 5)} \quad \text{and} \quad \underset{(x_2, y_2)}{(4, -1)} \quad \text{Label the points.}$$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{(-1) - (5)}{(4) - (-2)} = \frac{-6}{6} = -1$$

To apply the point-slope formula, use the slope, $m = -1$ and either given point. We will choose the point $(-2, 5)$ as (x_1, y_1).

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - 5 &= -1[x - (-2)] && \text{Substitute } m = -1,\ x_1 = -2, \text{ and } y_1 = 5. \\ y - 5 &= -1(x + 2) && \text{Simplify.} \\ y - 5 &= -x - 2 \\ y &= -x + 3 \end{aligned}$$

ProTip

TIP: The point-slope formula can be applied using either given point for (x_1, y_1). In Example 2, using the point $(4, -1)$ for (x_1, y_1) produces the same result.

$$y - y_1 = m(x - x_1)$$

$$y - (-1) = -1(x - 4)$$

$$y + 1 = -x + 4$$

$$y = -x + 3$$